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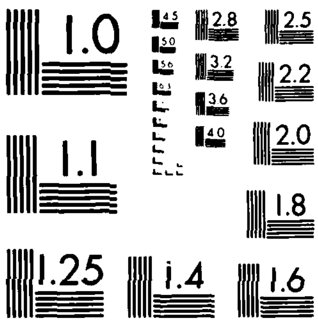

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report contains a summary of research concerning finite elasticity, phase transitions, viscoelasticity, viscoplasticity, plasticity, degenerate parabolic equations, and free boundaries.

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Uniqueness and Stability in

Nonlinear Elasticity and Viscoelasticity

Final Report

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January 14, 1985

U. S. Army Research Office

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Finite elasticity.

Motivated by papers of Ball [R1], Ericksen [R2,R3], James [R4], Knowles [R5], Knowles and Sternberg [R6], and Gurtin and Temam [R7], we have developed a general theory for two-phase deformations of elastic solids [P1]. Our main results concern the stability of such deformations. We proved that if a two-phase deformation is a local minimizer, then given any point p_0 of the surface of discontinuity, the piecewise-homogeneous deformation corresponding to the two values $F^{\pm}(p_0)$ of the deformation gradient $F(p_0)$ is a global minimizer. We showed further that the Eshelby conservation law, which is a direct consequence of the equilibrium equations for smooth deformations, is not generally valid for two-phase deformations; it is a necessary and sufficient condition that the Maxwell relation be satisfied, and is therefore a necessary condition for the stability of the deformation.

A theory of structured phase transformations, based on an elastic stored energy which depends on strain gradients (in addition to strain) is being developed in collaboration with Marshall Slemrod (RPI) and Jack Carr (Heriot-Watt University, Scotland). We have recently completed an analysis of the one-dimensional problem - for a filament of finite length - subjected to end forces or prescribed end displacements. Among our results we showed that for the force problem the only stable solutions

are those with constant strain (two phase solutions are unstable), while for the displacement problem, there are stable two phase solutions, but such solutions contain, at most, a single transition [P2,P3]. (See also the survey papers [P4,P5].) We also determined the properties of such solutions; the resulting analysis is quite difficult, involving a singular perturbation near a homoclinic orbit.

Viscoelasticity. Viscoplasticity.

In collaboration [P6] with L. F. Murphy (Oregon State University) we have considered the problem of minimizing the thermal stress induced during the cooling or heating of a thermorheologically simple[1] viscoelastic body. (Problems of this type occur, for example, during the curing of polymers.) We show - under certain simplifying assumptions - that optimal paths for such problems are canonical: they are independent of the shape of the body.

In a joint work [P7] with R. V. Browning (Los Alamos National Laboratory) we have presented a simplified constitutive equation[2] which describes the one-dimensional response of certain filled polymers. This constitutive equation isolates the rate-dependent portion of the stress response in the form of a convolution of a single stress-relaxation function with a more-or-less standard elastic-plastic stress-strain law.

[1] Cf., e.g., Muki and Sternberg [R8].

[2] This model is now being used - for the solution of actual problems - by the Los Alamos National Laboratory. Also, Lawrence Livermore Laboratory has developed (and coded) a three-dimensional theory [R9] based on our model.

To be precise, we introduced a "pseudo-stress" π which is related to the strain ε by a (rate-independent) elastic-plastic relation of the form

$$\pi = F(\varepsilon, \varepsilon_m),$$

where ε_m denotes the past maximum of strain. Then the stress σ is given by

$$\sigma(t) = \int_0^t G(t-s)\dot{\pi}(s)ds,$$

where G is a stress-relaxation function. The model has the virtue of simplicity, in that it models materials for which rate dependent behavior may be determined by a single stress relaxation test. The one-dimensional elastic-plastic function F is likewise easy to determine. We developed a straightforward process to determine these for three such polymers and found excellent fit to experimental data.

We have continued our work on stability in viscoelasticity; in particular, we have studied the buckling of a viscoelastic rod subject to suddenly applied arbitrarily-large axial end thrusts [P8]. In the literature one often sees the assertion that such problems have no solution. We show that this is true provided the loads are constant in time, but it is <u>not</u> true for certain time-dependent loadings. In particular, we show that:

(i) if the load starts at the instantaneous buckling load and initially decreases less rapidly than the normalized relaxation function, the rod will buckle;

(ii) if the load initially decreases more rapidly than the normalized relaxation function, the rod will not buckle.

The instantaneous buckling load is the buckling load calculated using the initial-value of the relaxation function. The above results demonstrate that the relaxation of a viscoelastic rod must be exploited in order to obtain buckled solutions.

Plasticity

We have developed [P9] a formulation of plasticity theory which utilizes a pair of hypoelastic stress-strain laws and a switching rule based on whether or not the current stress magnitude, $s(t)$, coincides with its past maximum

$$s_m(t) = \max_{\tau \leq t} s(\tau).$$

We do not find it necessary to introduce (a-priori) the notions of elastic and plastic strain or that of a yield surface. We prove that our formulation is, modulo certain assumptions, completely equivalent to classical plasticity with the von Mises yield condition and isotropic work-hardening: the existence of the yield surface and the use of plastic work as hardening parameter are consequences of the theory rather than initial assumptions.

Finally, the concepts underlying our formulation - namely, stress-rate, strain-rate, and the past maximum of stress - possibly make this formulation amenable to the numerical solution of boundary-value problems.

Degenerate parabolic equations and free boundaries.

In collaboration with R. C. MacCamy and E. A. Socolovsky (Carnegie-Mellon University) we have studied degenerate diffusion problems in which free boundaries occur, our major objective being the development of numerical procedures which effectively track these boundaries. In [P10] we introduced our ideas in terms of the one-dimensional porous media problem (cf., e.g., Aronson [R10], Caffarelli and Friedman [R11], Friedman [R12], Peletier [R13]), with initial data supported in a finite interval $[\alpha,\beta]$, an assumption which gives rise to a free boundary. We exploit the fact that particles move along trajectories determined by the flow velocity and, most importantly, that the free surface is itself such a trajectory. Writing $X(p,t)$ for the position at time t of the particle which at $t = 0$ occupied p, the porous medium problem is formally equivalent to an initial-value problem[1] for X on a fixed interval $\alpha \leq x \leq \beta$. Moreover, and most important, the curves

[1] Cf., Berryman [R14], whose work predates ours. Berryman also introduces Lagrangian coordinates (rather than the initial coordinate, Berryman takes $p = p(x,t)$ to be the total mass at t in the interval $(-\infty, x)$). Berryman's partial differential equation for X is simpler than ours, but his initial condition is more complicated.

$$x = X(\alpha,t), \qquad x = X(\beta,t)$$

form the free boundary.[1]

Our method of solving the porous medium problem is to solve the problem for X on the fixed interval $[\alpha,\beta]$. The price paid for this conceptual simplicity is that the equation for X is highly nonlinear, but this does not appear to be a serious concern in the numerical solution. Preliminary numerical studies, based on a simple difference scheme, seem to indicate that, even with a fairly crude mesh, the free boundary is tracked quite accurately; in fact, the accuracy seems to be $O(h^2)$, at least for concave initial data.

<u>General continuum mechanics</u>.

Experimentalists often report strain-data using logarithmic measures of strain, and these measures have been utilized extensively in the recent engineering literature. Whenever the principal axes of strain are fixed throughout the motion, it is known that the simple relationship

$$\underset{\sim}{D} = (\ln \underset{\sim}{V})^{\bullet}$$

holds, with $\underset{\sim}{D}$ the stretching tensor and $\underset{\sim}{V}$ the left stretch tensor. Previous work, however, failed to find an analogous formula in the general case. Some researchers have therefore

[1] This affords a method of studying the regularity of the free-boundary, a method subsequently used for that purpose by Höllig and Pilant [R15].

concluded that the strain measure $\ln \underset{\sim}{V}$ has very limited applicability. In [P11], written in collaboration with K. Spear (Georgia Tech), we established the precise circumstances in which the above formula holds. We also found a simple relationship between $\underset{\sim}{D}$ and the time derivative of $\ln \underset{\sim}{V}$ measured by an observer rotating with the axes of $\underset{\sim}{V}$ and showed that, for sufficiently small deformations, the Jaumann time-derivative of $\ln \underset{\sim}{V}$ is a good approximation to $\underset{\sim}{D}$.

Miscellaneous.

A paper [P12] was written discussing problems in continuum mechanics and population dynamics which are as yet unsolved, and which seem of interest from both mathematical and physical points of view. The first problem concerns the buckling of a viscoelastic, Euler-Bernoulli beam (cf. [P8]). The resulting system of equations forms a nonlinear eigenvalue problem with the buckled state dependent on the loading history. Based on our formulation, Puel and Mignot [R16] have developed a fairly complete solution.

The second problem is that of minimizing the residual stress that occurs during the cooling of a constrained viscoelastic material. What makes the problem interesting is the lack of convexity of the underlying cost function.

The third problem deals with the dispersal of age-dependent biological populations. Under certain simplifying assumptions the general system reduces to a pair of reaction-diffusion equations which are in some ways degenerate-parabolic. Existence for the

corresponding initial-value problem with compactly supported data has since been established by Hernandez [R17] in his thesis written under D. Aronson (University of Minnesota).

Participating Scientific Personnel

Kathleen Spear (who earned a Ph.D. degree while employed on the project.

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FINAL
REPORT OF INVENTIONS

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I hereby certify that, to the best of my knowledge and belief, no inventions, improvements or discoveries, which reasonably appear to be patentable, were conceived or first actually reduced to practice by persons engaged in the performance of the work under the above contract/grant during the period indicated, except as follows:

Name of Inventor:	Title of Invention

By

Title Professor of Mathematics

Date January 15, 1985

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